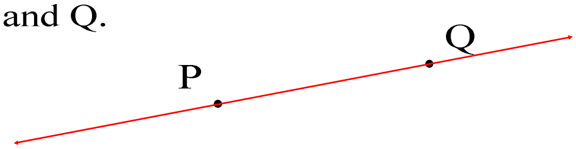
Euclid’s Postulates / Axioms

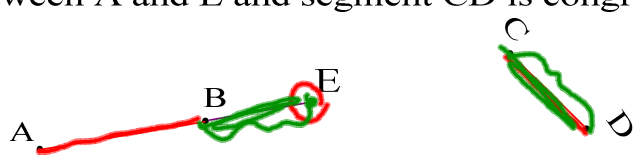
To draw conclusions and make arguments in mathematics, it is essential to take certain things as "givens" or postulates or axioms. These things are taken to be true without any proof, and it is these building blocks that are used to build up the rest of mathematics. In Euclidean Geometry, famous mathematician Euclid was the first mathematician to write down the five postulates or axioms that the rest of the mathematical world used in studying Geometry.

Euclid's Postulates:

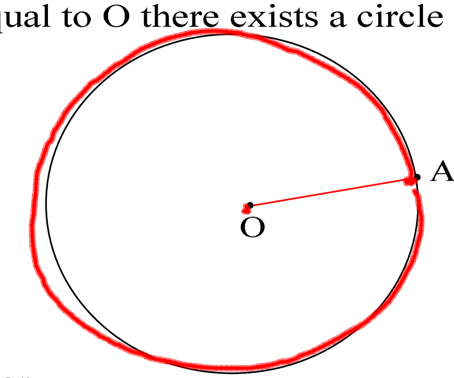
1. For every point P and every point Q not equal to P there exists a unique line that passes through P and Q.



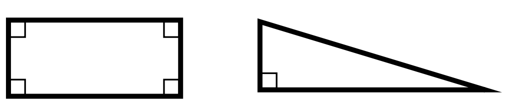
2. For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.



3. For every point O and every point A not equal to O there exists a circle with center O and radius OA.

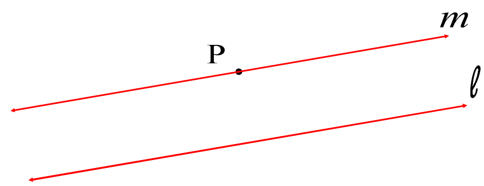


4. All right angles are congruent to each other.



Euclid's **5th** Postulate:

5. For every line ℓ and for every point P that does not lie on ℓ there exists a unique line *m* through P that is parallel to ℓ.



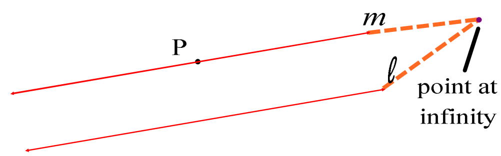
Euclid's fifth postulate cannot be proven using the other four postulates, though many mathematicians tried.

... because this postulate could not be proven using the other four, that meant that it was a necessary postulate in Euclid's Geometry.

Some mathematicians wondered if there could even be a consistent geometry that took the first four postulates as true, but assumed Euclid's fifth postulate to be false.

This could mean **two** things:

5. For every line ℓ and for every point P that does not lie on ℓ there does not exist~~s~~ a ~~unique~~ line m through P that is parallel to ℓ.

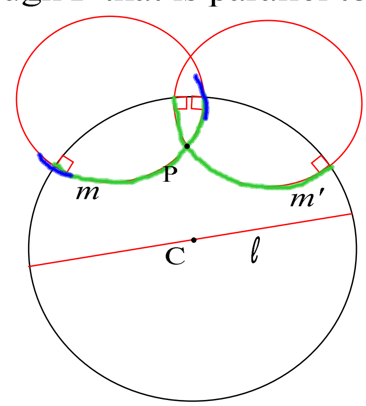


Elliptic Geometry

(Think of latitude or longitude lines on a globe. They are next to each other, yet they intersect at the poles.)

The sum of the angles of an elliptic triangle are always greater than 180 degrees. If you draw a triangle with points at the North Pole and two points along the equator, you will find that this triangle, with the curvature of the earth, has an angle measure sum of greater than 180 degrees, and up to 270°.

5. For every line ℓ and for every point P that does not lie on ℓ there exists more than one ~~a unique~~ line m through P that is parallel to ℓ.



Poincaré disk model of
Hyperbolic Geometry

Lines are either diameters of a disc or circles that intersect a disc at right angles.

The sum of the angles of a hyperbolic ~~elliptic~~ triangle are always less than 180 degrees.

You see, Euclidean Geometry assumes a flat plane. But there are many observable places in which we would like to do geometry in a curved domain, most notably, our earth and our universe.

For more on curved surfaces, multiple dimensions, and thought-provoking ponderings, read Flatland: A Romance of Many Dimensions by Edwin A. Abbott (1884) and Sphereland: A Fantasy About Curved Spaces and an Expanding Universe by Dionys Burger (1965). Also, Flatterland: Like Flatland, Only More So by Ian Stewart (2001).

Some Non-Euclidean Geometries:

Absolute geometry

Affine geometry

Elliptic geometry

Hyperbolic geometry

Projective geometry

Spherical geometry

Taxicab geometry