ArsDigitaUniversity

Month2:DiscreteMathematics   -ProfessorShaiSimonson

Examination2 –100points

Showallworkforpartialcredit.Youmayusetwohoursforthisexam.Afterone hour,raiseyourhandifyoufeelthatthetimeconstraintwill   betootight.

Name:_____

1.	**/30**
2.	**/28**
3.	**/28**
4.	**/14**

Total: **/100**

1. Sums (30 points)

a. Calculate a formula for the following sum, and prove your formula in any way you like. $(a+(a+1)+(a+2)+...+(a+b))$.

b. The following sum comes up in the analysis of certain sorting algorithms. Find a closed form expression for $1*2+2*4+3*8+4*16+...+n2^n$

2.(28points)ShortAnswers – Countability,Set s,Functions

a. Isthesetofallfinitebinarytreescountable?Explainbriefly.

b.Provethat $log(logx^{x})$is $\theta(logx)$.

c.Assumeadomainandrangeoftherealnumbers,andstatewhethereachofthe followingisafunction.Ifitis ,thenstatewhetheritisontoandwhetheritisone -one. Ifitisboth,thenwritedowntheinversefunction.

      i. x^{3}

      ii. $\sqrt{x}$

      ii.   Thefirstdigitof x afterthedecimalpoint

3. Towers Of Hanoi (28 points)

a. What happens when you try to run the following attempt at a solution to the Towers of Hanoi problem? Explain.

```
ToH( n,From,To,Using )    {
        if(n>0){
                ToH(n-1,From,Using,To  );
                ToH(1,From,To,Using  );
                ToH(n-1,Using,To,From  );
        }
}
```

A node in a Towers of Hanoi graph   is labeled 23221.

b. How many disks in this problem?

c. Draw a picture of the configuration represented by this node.

d. Is this a node that gets visited during the Hanoi solution from peg 1 to peg 2? Explain your answer.

4. ChineseRing sandGrayCodes(14points)

    a.   WhatarethetwoGraycodesthatprecedeandfollow101011?

    b.   Assumethat1meansthataringisaroundthesteelbar,and0means thattheringisfree.Does101011appearinthesequencethatsolves thesixringChine seRingPuzzle?Explainyouranswer.