ArsDigitaUniversity Month2:DiscreteMathematics -ProfessorShaiSimonson

ProblemSet1 –Logic,Proofs,andMathematicalReasoning

- 1. LogicProofs.
 - a. Prove hat $a \rightarrow b$ is equivalent to $\neg b \rightarrow \neg a$ using a truth table.
 - b. Proveitusing *algebraic*identities.
 - c. Prove hat $a \rightarrow b$ is not equivalent to $b \rightarrow a$.

2. Aristotle'sProofthattheSquareRootofTwoisIrrational.

- a. Prove the *lemma*, used by Aristotlein his proof, which says that if n^2 is even, so is *n*. (Hint: Remember that $a \rightarrow b$ is equivalent to $\neg b \rightarrow \neg a$).
- b. Provethatthesquarerootof3isirrationalusingAristotle'stechniques. Makesuretoprovetheappropriatelemma.
- c. If we use Aristotle's technique to *prove* the untrue assertion that the square root of 4 is irrational, where *exactly* is the hole in the proof?
- d. Using the fact that the square root of two is irrational, prove that sin($\pi/4$) is irrational.
- 3. InADU -ball, you can score 11 points for a goal, and 7 for an earmiss.
 - a. WriteaSchemeprogramthatprintsoutthenumberofgoalsandthe numberofnearmis sestoachieveagiventotalgreaterthan60.
 - b. Provethatyoucanachieveanyscoregreaterthan60.Thinkinductively and experiment.
- 4. Provebyinductionthat there are 2^{n} possible rows in a truth table with n variables.
- 5. IntherestroomofafancyItalia nrestaurantinMansfield,MA,thereisasignthat reads: *Pleasedonotleavevaluablesorlaptopcomputersinyourcar*. Assumingthatalaptopcomputerisconsideredavaluable,proveusingformal logic,thatthesentence *Pleasedonotleavevaluablesi nyourcar* isequivalentto thesignintherestroom.Provethat *Pleasedonotleavelaptopsinyourcar* isnot equivalent.
- 6. Prove that $a \mid b$, (*a*n and *b*), which is defined to be $\neg(a \land b)$, is complete. Write $(a \rightarrow b) \rightarrow b$ using just $|(n \text{ and }), then using just <math>\downarrow(n \text{ or })$.
- 7. Showhowtouseatruthtableinordertoconstructaconjunctivenormalformfor anyBooleanformula *W*.Hint:Considerthedisjunctivenormalformfor ¬*W*.
- 8. Euclidprovedthatthereareaninfinitenumberofprimes, by assuming that *n* is the high estprime, and exhibiting an umber that he proved must either be prime itself, or else have a prime factor greater than *n*. Write ascheme program to find the smallest *n* for which Euclid's proof does *not* provide an actual prime number.

- 9. Youhaveprovedbe forethatatruthtablewith n variableshas2 ⁿrows.
 - a. HowmanydifferentBooleanfunctionswith *n* variablesarethere?
 - b. For *n*=2,listallthefunctionsandidentifyasmanyasyoucanbyname.
- 10. Provebyinduction that for n>4, $2^n>n^2$.
- 11. Guessthenumber of different ways for *n* peopletoarrange themselves in a straightline, and provey our guess is correctly induction.
- 12. Uselogic with quantifiers and predicates to model the following three statements:

Allstudentsaretakingclasses.Somestudentsare notmotivated.Somepeople takingclassesarenotmotivated.

Prove, using resolution methods, that the third statement follows logically from the first two. (Reminder: You must take the conjunction of the first two statements and the negation of the hird, and derive a contradiction.)

- 13. ThefollowingalgebraicideaiscentralforKarnaughmaps.Karnaughmapsarea methodofminimizingthesizeofcircuitsfordigitallogicdesign.
 - a. Using algebraic manipulation, prove that the two Boolean formulae below are equivalent. (Hint: $x(a+\neg a)$ is equivalent to x.) $\neg yx + \neg zy + \neg xz$ and $\neg xy + \neg yz + \neg zx$
 - b. Verifyyourresultsusingatruthtable.
- 14. The exclusive -oroperator \oplus , is defined by the rule that $a \oplus b$ is true whenever a or *b* is true but not both.
 - a. Calculate $x \oplus x$, $x \oplus \neg x$, $x \oplus 1$, $x \oplus 0$.
 - b. Proveordisprove that $x+(y \oplus z) = (x+y) \oplus (x+z)$
 - c. Proveordisprove that $x \oplus (y+z) = (x \oplus y) + (x \oplus z)$
 - d. Writeconjunctivenormalformanddisjunctivenormalformformulaefor $x \oplus y$
 - e. The exclusive -or operatorismot *complete*. Which ones, if any, of the three operators { and, or, not } can be combined with exclusive -or to make a *complete* set.
- 15. The *n*thtrianglenumber T_n is defined to be the sum of the first *n* integers.
 - a. Provebyinduction that $T_n = n(n+1)/2$.
 - b. Provealg ebraically using (a), that $n^3 + (1+2+...+(n-1))^2 = (1+2+...(n-1)+n)^2$
 - c. Using(b)guessaformula for $1^{3}+2^{3}+3^{3}+\ldots+n^{3}$, and prove it by induction.
- 16. Guessaformulaforthesumbelow, and prove you are right by induction. $1+1(2)+2(3)+3(4)+\ldots+n(n+1)$