18.02 Multivariable Calculus
Fall 2007

# 18.02 Problem Set 2

Due Thursday 9/20/07, 12:45 pm.

**Part A** (15 points)

Hand in the <u>underlined</u> problems only; the others are for more practice.

Lecture 4. Thu Sept. 13 Theorems about square systems. Equations of planes
Read: Notes M.4 (pp. 9–10); Book pp. 798–800
Work: 1H/ <u>3abc</u>, 7; 12.4/ 23, 32, <u>50</u>, 51; 1E/ 1ab<u>cd</u>e, <u>2</u>, <u>6</u>.

Lecture 5. Fri Sept. 14 Parametric equations for lines and curves
Read: 12.4, pp. 796–797; 10.4 to top of p. 647
Work: 1E/ 3ab<u>c</u>, <u>4</u>, 5; 1I/ 1, 3<u>ab</u>d, <u>5</u>.

Lecture 6. Tue Sept. 18 Velocity, acceleration. Kepler's second law
Read: 12.5 to p. 808; (7) on p. 818; Notes K.
Work: 12.5/ 3, 4, 33; 1J/ 1, <u>2</u>, 3, <u>4</u>, 5, <u>6</u>, <u>9</u>, <u>10</u>; 1K/ <u>3</u>.

**Part B** (12 points)

**Directions:** Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous semesters. With each problem is the day it can be done.

Write the names of all the people you consulted or with whom you collaborated and the resources you used.

**Problem 1.** (Friday, 6 points: 3+3)

Consider the four points $A : (2, 4, 0)$, $B : (3, 1, 1)$, $C : (1, 1, 3)$, $D : (0, 5, 1)$. Find the distance between the lines $(AB)$ and $(CD)$, i.e. the distance between the closest points on these two lines, by using two different methods:

a) Find a pair of planes which are parallel to both lines, with the first plane containing the line $(AB)$ and the second plane containing the line $(CD)$. Then find the distance between these two planes.

b) Find parametric equations of the lines $(AB)$ and $(CD)$, and find the times at which the line segment connecting a point $P_1$ on $(AB)$ to a point $P_2$ on $(CD)$ is perpendicular to both lines. The length of this segment is then the distance between the lines.

*<u>Important:</u>* since the positions of the points $P_1$ and $P_2$ can be chosen independently of each other, the parameters on the two lines should be allowed to vary independently of each other (call them $t_1$ and $t_2$ for example).

**Problem 2.** (Friday, 6 points: 4+2)

A mechanical device consists of two circular gears, one of radius 2 centered at $(0, -2)$ and the other of radius 1 centered at $(0, 1)$. The gear of radius 2 rotates clockwise at unit angular velocity (1 radian per second), while the gear of radius 1 rotates counterclockwise without slipping at the contact point. The two gears each carry a small peg on their circumference, and these pegs are connected together by an elastic band. Initially, both pegs are at the origin.

a) Write parametric equations for the motion of the midpoint $P$ of the elastic band. (Use vectors; begin by expressing the position vector in terms of simpler vectors, then express each of them in terms of $t$).

b) (Tuesday) Express the velocity of the point $P$ as a function of $t$. Calculate the velocity and the acceleration at $t = 0$.