Recitation 3, February 14, 2006

Numerical methods; Linear models

1. Use Euler's method to estimate the value at $x = 1.5$ of the solution of $y' = y^2 - x^2 = F(x, y)$ at with $y(0) = -1$. Use $h = 0.5$. Make the table: k, x_k,y_k, $A_k = F(x_k, y_k)$, $h \cdot A_k$. Draw the Euler polygon.

2. Is the estimate from **1.** too large or to small?

3. Here's a "mixing problem." A tank holds V liters of salt water. Suppose that a saline solution with concentration of c gm/liter is added at the rate of r liters/minute. A mixer keeps the salt essentially uniformly distributed in the tank. A pipe lets solution out of the tank at the same rate of r liters/minute. Write down the differential equation for the *amount* of salt in the tank. [Not the concentration!] Check the units in your equation! Write it in standard form.

4. Now assume that c and r are constant; in fact, assume that $V = 1$ and $r = 2$. Solve this equation, under the assumption that $x(0) = 0$

What is the limiting amount of salt in the tank? Does your result jibe with simple logic? When will the tank contain half that amount?

5. Now suppose that the out-flow from this tank leads into another tank, also of volume 1, and that at time $t = 0$ the water in it has no salt in it. Again there is a mixer and an outflow. Write down a differential equation for the amount of salt in this second tank, as a function of time.